EECS 442 Discussion

Arash Ushani

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Projective Geometry

- For more detail, see HZ Chapter 2

Representing Lines

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$$(ka)x + (kb)y + kc = 0$$
$$\mathbf{l} = (ka, kb, kc)^\top$$

- Equivalance Classes

Points on Lines

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$$\mathbf{x} = (x, y, 1)^\top$$
$$\mathbf{x}^\top \mathbf{l} = 0$$
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HW Hints: Lines and transformations

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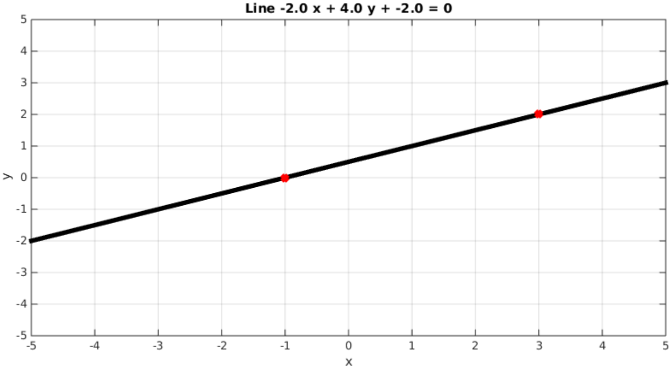
- Alernatively, if $\mathbf{l}^\top \mathbf{x} = 0$, what can you say about $\mathbf{l}^\top \mathbf{H}^{-1}\mathbf{Hx}$ if $\mathbf{H}$ is a projective transformation?

MATLAB Exercise

- Go to CTools → Resources → Discussion → 09_23_matlab.zip



Direct Linear Transform

- For more detail, see chapter 4 section 1 in HZ

Direct Linear Transform

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- We want to find the transformation $\mathbf{H}$ such that $\mathbf{x}'_i = \mathbf{H}\mathbf{x}_i$

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- If $\mathbf{x}'_i = (x'_i, y'_i, w'_i)^\top$, then:

$$\mathbf{x}'_i \times (\mathbf{H}\mathbf{x}_i) = \begin{bmatrix} y'_i\mathbf{h}^{3\top}\mathbf{x}_i - w'_i\mathbf{h}^{2\top}\mathbf{x}_i \\ w'_i\mathbf{h}^{1\top}\mathbf{x}_i - x'_i\mathbf{h}^{3\top}\mathbf{x}_i \\ x'_i\mathbf{h}^{2\top}\mathbf{x}_i - y'_i\mathbf{h}^{1\top}\mathbf{x}_i \end{bmatrix} = \mathbf{0}$$

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$$\begin{bmatrix} \mathbf{0}^\top & -w'_i\mathbf{x}_i^\top & y'_i\mathbf{x}_i^\top \\ w'_i\mathbf{x}_i^\top & \mathbf{0}^\top & -x'_i\mathbf{x}_i^\top \\ -y'_i\mathbf{x}_i^\top & x'_i\mathbf{x}_i^\top & \mathbf{0}^\top \end{bmatrix} \begin{bmatrix} \mathbf{h}^1 \\ \mathbf{h}^2 \\ \mathbf{h}^3 \end{bmatrix} = \mathbf{0}$$

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- With 4 correspondences, we have $\mathbf{Ah} = \mathbf{0}$ where $\mathbf{A}$ is rank 8

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- SVD Decomposition:

$$\mathbf{A} = \mathbf{UDV}^\top$$

- $\mathbf{D}$ is a diagonal matrix of the singular values of $\mathbf{A}$
- $\mathbf{V}$ contains the singular vectors of $\mathbf{A}$ as column vectors

Next Week

- Guest Speaker: Steven Parkison, Calibration Expert
- No Office Hours